Connes amenability of l^1-Munn algebras

Ali Ghaffari, Samaneh Javadi and Ebrahim Tamimi

Abstract. In this paper, we study Connes amenability of l^1-Munn algebras. We apply this results to semigroup algebras. We show that for a weakly cancellative semigroup S with finite idempotents, amenability and Connes amenability are equivalent.

Keywords. Amenability, Banach algebras, derivation, l^1-Munn algebras,semigroup algebras

1   Introduction

In [4], Eslamzadeh introduced l^1-Munn algebras. He used these algebras to characterize amenable semigroup algebras. A special case of these algebras was introduced by Munn [18]. l^1-Munn algebras has been studied in some texts. In [1], Blackmore showed the l^1-Munn algebra of the group algebra $l^1(G)$ is weakly amenable. Eslamzadeh in [5] and [6] investigated the structure of l^1-Munn algebras. Duncan and Paterson used the l^1-Munn algebras to study of semigroup algebras of completely simple semigroups [3].

The motivation to study of the theory of amenable von Neumann algebras stems from the fact that they are dual. In [12], it is shown that if $\mathcal{A}$ is a von Neumann algebra containing a weak*-dense amenable C^*-subalgebra, then for every normal Banach $\mathcal{A}$-bimodule E, every weak*-continuous derivation $D : \mathcal{A} \to E$ is inner. This concept of amenability was called Connes amenability [9]. In [21], Runde extended the notion of Connes-amenability to dual Banach algebras. For a locally compact group G, the group algebra $l^1(G)$ and the measure algebra $M(G)$ are two examples of dual Banach algebras. In [23], Runde introduced normal, virtual diagonals for a dual Banach algebra and showed that the existence of a normal virtual diagonal for $M(G)$ is equivalent to it being Connes amenable. Also in [22], it is shown that G is amenable if and only if $M(G)$ is Connes amenable. In particular, $l^1(G)$ is amenable if and only if $l^1(G)$ is Connes amenable.

The investigation of Connes amenability for dual Banach algebras which are not von Neumann algebra is interesting for many authors, see [24], [2] and [7]. Several authors have generalized the earlier concept of amenability introduced by Lau in [13] (see [14], [15], [16] and [17]). Recently the authors have introduced the ϕ-version of Connes amenability of dual Banach algebra